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Noh–Fougères–Mandel operational phase operator as an entangling operator and the corresponding squeezed state

Hongyi Fan[1,2] **and Zhihu Sun**[2]

[1] CCAST (World Laboratory), PO Box 8730, Beijing 100080, People's Republic of China
[2] Department of Material Science and Engineering, University of Science and Technology of China, Hefei, Anhui 230026, People's Republic of China

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Abstract

We construct a new number-difference and operational phase entangled state by operating the Noh–Fougères–Mandel (NFM) phase operator on the two-mode twin-photon state. Its Schmidt decomposition is derived by virtue of the previously constructed Einstein–Podolsky–Rosen eigenstate. We reach the conclusion that the NFM phase operator is essentially an entangling operator, based on which a new type of number-difference and operational phase squeezed state can be introduced.

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1. Introduction

Recently entangled states have been applied to discussing quantum computation, quantum teleportation, quantum cryptography and quantum superdense coding [1–3]. In an entangled quantum state, a measurement performed on one part of the system provides information on the remaining part, as first pointed out by Einstein, Podolsky and Rosen (EPR) [4] in their famous paper arguing the incompleteness of quantum mechanics. EPR introduced the EPR wavefunction, the common eigenfunction of two particles' relative position $X_1 - X_2$ (with centre of mass coordinate x_0) and their total momentum $P_1 + P_2$ (with eigenvalue $p_0 = 0$)

$$\psi(x_1, p_{1;}x_2, p_2) = \delta(x_1 - x_2 + x_0)\delta(p_1 + p_2) \qquad (1)$$

which describes a sharply correlated two-particle system. For example, if one measures the momentum of particle 1 and finds $p_1 = k$, then the outcome of a subsequent measurement of momentum on particle 2 is $p_2 = -k$ with certainty. Thus there is a mysterious nonlocal entanglement between separated quantum objects. The EPR argument has stimulated many discussions on the nonlocality and entanglememt inherent in quantum mechanics. Remarkably, the simultaneous eigenstate $|\eta\rangle$ of commuting operators $(X_1 - X_2, P_1 + P_2)$ in two-mode Fock space can be explicitly constructed [5]: it is

$$|\zeta\rangle = \exp[-\tfrac{1}{2}|\zeta|^2 + \zeta a_1^\dagger - \zeta^* a_2^\dagger + a_2^\dagger a_1^\dagger]|00\rangle \qquad \zeta = \frac{1}{\sqrt{2}}(\zeta_1 + \mathrm{i}\zeta_2). \qquad (2)$$

On the other hand, the common eigenvector of $(X_1 + X_2, P_1 - P_2)$ is

$$|\eta\rangle = \exp[-\tfrac{1}{2}|\eta|^2 + \eta a_1^\dagger + \eta^* a_2^\dagger - a_2^\dagger a_1^\dagger]|00\rangle \tag{3}$$

where $\eta = \frac{1}{\sqrt{2}}(\eta_1 + \mathrm{i}\eta_2)$ is a complex number, $|00\rangle$ is the two-mode vacuum state and $(a_i, a_i^\dagger)$, $i = 1, 2$, are two-mode Bose annihilation and creation operators in Fock space related to (X_i, P_i) by

$$X_i = \frac{1}{\sqrt{2}}(a_i + a_i^\dagger) \qquad P_i = \frac{1}{\sqrt{2}\mathrm{i}}(a_i - a_i^\dagger). \tag{4}$$

The $|\eta\rangle$ state obeys the eigenvector equations

$$(a_1 + a_2^\dagger)|\eta\rangle = \eta|\eta\rangle \qquad (a_2 + a_1^\dagger)|\eta\rangle = \eta^*|\eta\rangle. \tag{5}$$

It then follows from (4) and (5) that

$$(X_1 + X_2)|\eta\rangle = \eta_1|\eta\rangle \qquad (P_1 - P_2)|\eta\rangle = \eta_2|\eta\rangle. \tag{6}$$

As $|\eta\rangle$ is qualified to make up a new quantum mechanical representation, we name it the entangled state representation with a continuous variable, not only because $|\eta\rangle$ satisfies the completeness relation [5]

$$\int \frac{\mathrm{d}^2\eta}{\pi}|\eta\rangle\langle\eta| = 1 \qquad \mathrm{d}^2\eta \equiv \tfrac{1}{2}\,\mathrm{d}\eta_1\,\mathrm{d}\eta_2 \tag{7}$$

and possesses the orthonormal property

$$\langle\eta'|\eta\rangle = \pi\delta(\eta - \eta')\delta(\eta^* - \eta'^*) \tag{8}$$

but also because the two-mode squeezing operator has its natural representation in the $\langle\eta|$ representation [6]

$$\mu\int \frac{\mathrm{d}^2\eta}{\pi}|\eta\mu\rangle\langle\eta| = \exp[f(a_1^\dagger a_2^\dagger - a_1 a_2)] \qquad \mu = \exp f \tag{9}$$

and the two-mode squeezed state itself is an entangled state which entangles the idler mode and signal mode as an outcome of a parametric-down conversion process [7]. In [8] the basic ingredient of the $|\eta\rangle$ state about the coordinate-momentum entanglement is demonstrated through its Schmidt decomposition process [9]; i.e., we perform the following Fourier integration:

$$\begin{aligned}\int_{-\infty}^{\infty}\frac{\mathrm{d}\eta_2}{2\pi}\left|\eta = \frac{1}{\sqrt{2}}(\eta_1 + \mathrm{i}\eta_2)\right\rangle \mathrm{e}^{-\mathrm{i}u\eta_2} \\ &= \pi^{-\frac{1}{2}}\exp\left[-\frac{1}{4}\eta_1^2 - u^2 + \sqrt{2}\left(u + \frac{\eta_1}{2}\right)a_1^\dagger + \sqrt{2}\left(\frac{\eta_1}{2} - u\right)a_2^\dagger - \frac{a_1^{\dagger 2} + a_2^{\dagger 2}}{2}\right]|00\rangle \\ &= \left|u + \frac{\eta_1}{2}\right\rangle_1 \otimes \left|\frac{\eta_1}{2} - u\right\rangle_2\end{aligned} \tag{10}$$

where

$$|u\rangle_i = \pi^{-\frac{1}{4}}\exp\left[-\frac{1}{2}u^2 + \sqrt{2}ua_i^\dagger + \frac{a_i^{-\dagger 2}}{2}\right]|0\rangle_i \tag{11}$$

is the coordinate eigenvector. The inverse Fourier transformation of (10) is

$$|\eta\rangle = \mathrm{e}^{-\mathrm{i}\eta_1\eta_2/2}\int_{-\infty}^{\infty}\mathrm{d}u\,|u\rangle_1 \otimes |\eta_1 - u\rangle_2 \mathrm{e}^{\mathrm{i}u\eta_2} \tag{12}$$

which indicates that $|\eta\rangle$ is really an entangled state. Therefore we name $|\eta\rangle$ state the EPR entangled eigenstate.

Noh, Fougères and Mandel (NFM) proposed an operational quantum phase description [10], which means that the Hermitian phase operators can be defined as observables in an operational way. They started by analysing what is usually measured in an experiment and then introduced operators that represented the measurement, based on the correspondence with classical optics. As measurement always involves the difference between two phases, and as an interference or homodyne experiment usually yields the cosine and sine of the phase difference, they introduced measured operators for the cosine and sine of the phase difference that corresponded to a particular measurement scheme—the eight-port homodyne interferometer—by replacing the classical light intensities at the equipment's four detectors by number operators, and in the strong-local-oscillator limit the NFM phase operators become the cosine and sine of the phase difference between two classical electromagnetic fields in the two input modes. They can therefore serve as a reasonable definition for the sine and cosine of the phase difference between a coherent state (generated by a local oscillator) and the signal state. NFM also gave a scheme of how to calculate expectation values of a function of their phase operators. Later, in [11], Freyberger *et al* further showed that in the limit of a strong local oscillator, the NFM description contains an essential two-mode basis which leads to the simultaneously measurable operator pair. It also provides the NFM operational phase operator in the following form [12]:

$$\sqrt{\frac{a_1 + a_2^\dagger}{a_1^\dagger + a_2}} \equiv \mathrm{e}^{\mathrm{i}\Phi} \qquad \sqrt{\frac{a_1^\dagger + a_2}{a_1 + a_2^\dagger}} \equiv \mathrm{e}^{-\mathrm{i}\Phi} \qquad \cos\Phi = (\mathrm{e}^{\mathrm{i}\Phi} + \mathrm{e}^{-\mathrm{i}\Phi})/2 \tag{13}$$

with a natural representation because from (5) and (13) we see that in the $\langle\eta|$ representation $\mathrm{e}^{\mathrm{i}\Phi}$ behaves as [12]

$$\mathrm{e}^{\mathrm{i}\Phi} = \int \frac{\mathrm{d}^2\eta}{\pi} \mathrm{e}^{\mathrm{i}\varphi}|\eta\rangle\langle\eta| \qquad \mathrm{e}^{\mathrm{i}\varphi} = \left(\frac{\eta}{\eta^*}\right)^{\frac{1}{2}} \tag{14}$$

manifestly exhibiting its phase behaviour. Note that $[a_1 + a_2^\dagger, a_1^\dagger + a_2] = 0$, so they can reside in the same square root. $\mathrm{e}^{\mathrm{i}\Phi}$ is a unitary operator. By introducing the two-mode photon number-difference operator $D \equiv a^\dagger a - b^\dagger b$, it has been shown by Fan and Xiao [13] that

$$[\mathrm{e}^{\mathrm{i}\Phi}, D] = \mathrm{e}^{\mathrm{i}\Phi} \qquad [\mathrm{e}^{-\mathrm{i}\Phi}, D] = -\mathrm{e}^{-\mathrm{i}\Phi}. \tag{15}$$

Therefore, the number-difference operator and NFM operational phase operator $\mathrm{e}^{\mathrm{i}\Phi}$ can be regarded as a pair of conjugate operators. This thus challenges us with the following question: can we use the NFM operational phase operator to construct a new kind of entangled state which implicitly involves number-difference and operational phase entanglement? Noting that the two-mode squeezed vacuum state (an entangled state) is

$$\begin{aligned}\exp[f(a_1^\dagger a_2^\dagger - a_1 a_2)]|0,0\rangle &= (\cosh f)^{-1}\exp[a_1^\dagger a_2^\dagger \tanh f]|0,0\rangle \\ &= (\cosh f)^{-1}\sum_{n=0}\tanh^n f|n,n\rangle\end{aligned} \tag{16}$$

where $|n,n\rangle = (a_1^\dagger a_2^\dagger)^n/n!|0,0\rangle$ is a twin-photon state, we are naturally led to consider operating with the NFM phase operator on the twin-photon state.

2. Construction of number-difference and operational phase entangled state and its Schmidt decomposition

We construct a new entangled state by operating

$$(\mathrm{e}^{\mathrm{i}\Phi})^q|m,m\rangle \equiv \|q,m\rangle \tag{17}$$

where q is an integer, which could be negative. Because $[D, (a_1 + a_2^\dagger)(a_1^\dagger + a_2)] = 0$, one has

$$[D, \mathrm{e}^{\mathrm{i}q\Phi}] = -q\mathrm{e}^{\mathrm{i}q\Phi} \qquad [D, \mathrm{e}^{-\mathrm{i}\Phi}] = \mathrm{e}^{-\mathrm{i}\Phi}; \tag{18}$$

this state $\|q, m\rangle$ is the eigenstate of the two-mode number-difference operator,

$$D\|q, m\rangle = D\mathrm{e}^{\mathrm{i}q\Phi}|m, m\rangle = [D, \mathrm{e}^{\mathrm{i}q\Phi}]|m, m\rangle = -q\|q, m\rangle. \tag{19}$$

By introducing the two-variable Hermite polynomial

$$H_{m,n}(\eta, \eta^*) = \sum_{l=0}^{\min(m,n)} \frac{m!n!}{l!(m-l)!(n-l)!}(-1)^l \eta^{m-l}\eta^{*(n-l)} = H_{m,n}(r, r)\mathrm{e}^{\mathrm{i}(m-n)\theta} \tag{20}$$

and its generating function [5, 14]

$$\exp[-tt' + \eta t + \eta^* t'] = \sum_{m,n=0}^{\infty} \frac{t^m t'^n}{m!n!} H_{m,n}(\eta, \eta^*) \tag{21}$$

$|\eta\rangle$ can be expressed in two-mode Fock space as

$$|\eta\rangle = \sum_{m,n=0}^{\infty} \mathrm{e}^{-\frac{1}{2}|\eta|^2} \frac{1}{\sqrt{m!n!}} H_{m,n}(\eta, \eta^*)|m\rangle_1|n\rangle_2. \tag{22}$$

The explicit form of $\|q, m\rangle$ in two-mode Fock space can be deduced as follows. Using equations (14) and (22) we see

$$\begin{aligned}\mathrm{e}^{\mathrm{i}\Phi q}|m\rangle_1|m\rangle_2 &= \int \frac{\mathrm{d}^2\eta}{\pi}\mathrm{e}^{\mathrm{i}q\varphi}|\eta\rangle\langle\eta|m\rangle_1|m\rangle_2 \\ &= \sum_{m',n'=0}^{\infty} \int_0^{2\pi} \frac{\mathrm{d}\varphi}{2\pi}\mathrm{e}^{\mathrm{i}(m'-n'+q)\varphi} \int_0^{\infty} \mathrm{d}(r^2)\mathrm{e}^{-r^2} \frac{H_{m',n'}(r,r)H_{m,m}(r,r)}{\sqrt{m'!n'!m!}}|m'\rangle_1|n'\rangle_2.\end{aligned} \tag{23}$$

Then according to the definition of the associated Laguerre polynomial $L_n^\mu(x) = \sum_{k=0}^{n}\binom{n+\mu}{n-k}\frac{(-x)^k}{k!}$ [14] and equation (20), it is not difficult to see that

$$H_{m',n'}(r, r) = p'!(-1)^{p'}r^{l'}L_{p'}^{l'}(r^2) \qquad H_{m,m}(r, r) = (-1)^m m! L_m^0(r^2) \qquad \eta = r\mathrm{e}^{\mathrm{i}\varphi} \tag{24}$$

where

$$p' = \min(m', n') \qquad l' = |m' - n'|. \tag{25}$$

Therefore, using the integration formula [14]

$$\int_0^\infty \mathrm{d}x\, \mathrm{e}^{-x}x^\lambda L_n^\mu(x)L_{n'}^{\mu'}(x) = (-1)^{n+n'}\Gamma(\lambda+1)\sum_{k=0}^{\min(n,n')}\binom{\lambda-\mu}{n-k}\binom{\lambda-\mu'}{n'-k}\binom{\lambda+k}{k} \tag{26}$$

where $\binom{\alpha}{\beta} \equiv \frac{\alpha(\alpha-1)\cdots(\alpha-\beta+1)}{\beta!}$ and $\Gamma(x)$ is the gamma function, and the relation between the Laguerre polynomial and the two-variable Hermite polynomial (24), we obtain

$$\begin{aligned}\int_0^\infty \mathrm{d}(r^2)\,\mathrm{e}^{-r^2}\frac{H_{m',n'}(r,r)H_{m,m}(r,r)}{\sqrt{m'!n'!m!}} &= (-1)^{p'+m}\frac{p'!}{\sqrt{m'!n'!}}\int_0^\infty \mathrm{d}x\,\mathrm{e}^{-x}x^{\frac{l'}{2}}L_{p'}^{l'}(x)L_m^0(x) \\ &= \frac{p'!\Gamma(\frac{l'}{2}+1)}{\sqrt{m'!n'!}}\sum_{k=0}^{\min(p',m)}\binom{-\frac{l'}{2}}{p'-k}\binom{\frac{l'}{2}}{m-k}\binom{\frac{l'}{2}+k}{k}.\end{aligned} \tag{27}$$

Inserting (27) into (23) leads to

$$\|q, m\rangle = \sum_{m',n'=0}^{\infty}\sum_{k=0}^{\min(p',m)}\delta_{m',n'-q}\frac{p'!\Gamma(\frac{l'}{2}+1)}{\sqrt{m'!n'!}}\binom{-\frac{l'}{2}}{p'-k}\binom{\frac{l'}{2}}{m-k}\binom{\frac{l'}{2}+k}{k}|m'\rangle_1|n'\rangle_2. \tag{28}$$

Thus for $q \geqslant 0$ we obtain the explicit expansion form of $e^{iq\Phi}|m\rangle_1|m\rangle_2$ in two-mode Fock space

$$\|q,m\rangle = e^{iq\Phi}|m\rangle_1|m\rangle_2 = \Gamma\left(\frac{q}{2}+1\right) \times \sum_{m'=0}^{\infty}\sum_{k=0}^{\min(m',m)} \sqrt{\frac{m'!}{(m'+q)!}} \begin{pmatrix} -\frac{q}{2} \\ m'-k \end{pmatrix} \begin{pmatrix} \frac{q}{2} \\ m-k \end{pmatrix} \begin{pmatrix} \frac{q}{2}+k \\ k \end{pmatrix} |m'\rangle_1|m'+q\rangle_2. \quad (29)$$

Equation (29) reveals an interesting property of the phase operator $e^{i\Phi}$, i.e. $e^{iq\Phi}$ acting on a twin-photon state $|m\rangle_1|m\rangle_2 \equiv |m,m\rangle$ yields the superposition of an infinite number of two-mode Fock states, each state's idler-mode photon number being larger than its signal mode by q. This is just the Schmidt decomposition of $\|q,m\rangle$ which demonstrates its entanglement property between the two modes. For example, when $m=0$, $q=1$, we have

$$e^{i\Phi}|0\rangle_1|0\rangle_2 = \Gamma\left(\frac{3}{2}\right)\sum_{m'=0}^{\infty}\frac{1}{\sqrt{(m'+1)}}\begin{pmatrix} -\frac{1}{2} \\ m' \end{pmatrix}|m'\rangle_1|m'+1\rangle_2. \quad (30)$$

It then follows that

$${}_2\langle 0|_1\langle 1|R|0\rangle_1|0\rangle_2 = \Gamma(\tfrac{3}{2}). \quad (31)$$

It is not difficult to prove that the $\|q,m\rangle$ state set spans a complete and orthogonal set, i.e.

$$\sum_{q=-\infty}^{\infty}\sum_{n=0}^{\infty}\|q,m\rangle\langle q,m\| = 1 \quad (32)$$

$$\langle q',m'\|q,m\rangle = \delta_{q,q'}\delta_{m,m'} \quad (33)$$

which coincides with the unitarity of the NFM phase operator.

3. Application of $\|q,m\rangle$ in constructing the number-difference-operational phase squeezed state

Due to (15), we have

$$e^{i\Phi}De^{-i\Phi} = D+1 \qquad [e^{i\Phi}D, De^{-i\Phi}] = 2D+1 \qquad [e^{i\Phi}D, D] = De^{i\Phi}; \quad (34)$$

hereafter for convenience we adopt the notation $K_- \equiv e^{i\Phi}D$, $K_+ \equiv De^{-i\Phi}$ and $K_0 \equiv D+\frac{1}{2}$. Equation (34) indicates that K_-, K_+ and K_0 constitute an $SU(1,1)$ Lie algebra,

$$[K_-, K_+] = 2K_0 \qquad [K_0, K_\pm] = \pm K_\pm. \quad (35)$$

The Casimir operator is

$$C = K_0^2 - \tfrac{1}{2}(K_+K_- + K_-K_+) = -\tfrac{1}{4}. \quad (36)$$

The twin-photon state $|l,l\rangle$ is a state annihilated by K_-, $K_-|l,l\rangle = 0$. According to the procedures of constructing coherent states associated with Lie algebra [15], the Casimir operator acting on $|l,l\rangle$ should satisfy

$$C|l,l\rangle = -\tfrac{1}{4}|l,l\rangle = k(k-1)|l,l\rangle \quad (37)$$

which indicates that the Bargmann index is $k=\frac{1}{2}$, $K_0|l,l\rangle = \frac{1}{2}|l,l\rangle$, and the twin-photon state $|l,l\rangle$ is the minimum-value state. We now introduce the unitary operator

$$U = \exp[\zeta K_+ - \zeta^* K_-)] \quad (38)$$

and construct a new squeezed-like state as

$$U|l,l\rangle = \exp[\zeta K_+ - \zeta^* K_-)]|l,l\rangle \quad (39)$$

where the complex parameter $\zeta = g\mathrm{e}^{\mathrm{i}\theta}$, $(g \geqslant 0)$. Since K_-, K_+ and K_0 obey the $SU(1,1)$ Lie algebra, U is disentangled as

$$U = \exp[K_+\mathrm{e}^{\mathrm{i}\theta}\tanh g]\exp[-2K_0\ln\cosh g]\exp[-\mathrm{e}^{-\mathrm{i}\theta}K_-\tanh g]. \tag{40}$$

Thus

$$U|l,l\rangle = (\cosh g)^{-1}\exp[K_+\mathrm{e}^{\mathrm{i}\theta}\tanh g]|l,l\rangle = (\cosh g)^{-1}\sum_{n=0}^{\infty}\frac{1}{n!}(K_+\mathrm{e}^{\mathrm{i}\theta}\tanh g)^n|l,l\rangle. \tag{41}$$

It follows from equations (17) and (18) that

$$D\mathrm{e}^{-\mathrm{i}n\Phi}|l,l\rangle = n\mathrm{e}^{-\mathrm{i}n\Phi}|l,l\rangle = n\|-n,l\rangle \tag{42}$$

therefore $(K_+)^n|l,l\rangle = (D\mathrm{e}^{-\mathrm{i}\Phi})^n|l,l\rangle = n!\mathrm{e}^{-\mathrm{i}n\Phi}|l,l\rangle = n!\|-n,l\rangle$; this substituted into equation (41) gives

$$U|l,l\rangle = (\cosh g)^{-1}\sum_{n=0}^{\infty}(\mathrm{e}^{\mathrm{i}\theta}\tanh g)^n\mathrm{e}^{-\mathrm{i}\Phi n}|l,l\rangle = (\cosh g)^{-1}\sum_{n=0}^{\infty}(\mathrm{e}^{\mathrm{i}\theta}\tanh g)^n\|-n,l\rangle \tag{43}$$

which in form is very like the expression of the two-mode squeezed vacuum state in two-mode Fock space (16). This is not a coincidence, since they are both the Perelomov $SU(1,1)$ coherent state [15]. Here $\|-n,l\rangle$ is the new entangled state whose entanglement originates from the entangled operator—the NFM phase operator. Up to now, we have expanded $U|l,l\rangle$ in the entangled state space spanned by $\|-n,l\rangle$. The probability of $U|l,l\rangle$ occupancy in $\|-n,l\rangle$ is

$$|\langle -n,l\|U|l,l\rangle|^2 = (\cosh g)^{-2}\tanh^{2n} g = (1-x)x^n \qquad x = \tanh^2 g \tag{44}$$

which is a geometric distribution.

4. Discussion

In the above discussions we have provided a new entangled state $\|q,m\rangle$ and its corresponding squeezed state (43). (Note that the two-mode squeezed state (16) is approximately a quadrature EPR state [8] and can be used for teleportation.) In a very recent paper by Milburn and Braunstein [16], the teleportation using number and phase measurements is discussed, where the target state $|\psi\rangle_\mathrm{T}$, which is going to be sent by Alice to Bob, is expanded in the photon number basis as

$$|\psi\rangle_\mathrm{T} = \sum_{m=0}^{\infty} c_m|m\rangle_\mathrm{T} \tag{45}$$

and the input state to the receiver and sender is

$$|\psi\rangle_\mathrm{in} = (1-\lambda^2)^{1/2}\sum_{n,m=0}^{\infty}\lambda^n c_m|m\rangle_\mathrm{T}\otimes|n\rangle_\mathrm{Alice}\otimes|n\rangle_\mathrm{Bob}. \tag{46}$$

To facilitate the description of the joint measurements that need to be made on T and A at the receiver, they introduced the eigenstates $|\rangle_{\mathrm{A,T}}$ of the number-difference operator $N_\mathrm{T} - N_\mathrm{A}$ for modes T and A. Now our new state $\|q,m\rangle_{\mathrm{A,T}}$ just fits this demand, since it is an eigenstate of the number-difference operator. Or one can use our new entangled state as a EPR source for Alice and Bob to share, as $N_\mathrm{B} - N_\mathrm{A}$ is definite. For further detailed theoretical analysis for teleportation using number and phase measurements or using a squeezed state one may refer to [16].

5. Conclusion

In this paper we have demonstrated that NFM phase operator (a unitary operator) plays the role of entanglement between the two photon modes in its special way and is thus an entangling operator. Based on this, the new number-difference phase squeezed state can be constructed on the solid foundation of $SU(1, 1)$ Lie algebra, which also manifestly exhibits entanglement.

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